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# THE FIRST ATTEMPT AT A TABLE OF INTEGRALS.

By NOAH R. BRYAN, University of Maine

There is apparent an ever-growing interest in the calculus in the fields of applied mathematics and of secondary education. In the field of applied mathematics, we find that an author of an elementary book on wireless telegraphy has considered it necessary to include a chapter on the calculus;[1] we find that chemists no longer look askance at the introduction of formal mathematics into their subject but are beginning to welcome the use of more calculus in their science;[2] and, since today the aëroplane is built according to the knowledge obtained by the careful methods of the laboratory, it is only natural to find its construction described in the language of the calculus.[3] In the field of secondary education, a study of the status of mathematics in the curricula in schools abroad reveals the fact that in ten of these countries some calculus is taught before the end of the year which corresponds to our twelfth school year or the last year in high school.[4] Also, not only has it been recommended that an elective course in the simpler parts of the calculus should be offered in our first-class high schools,[5] but such a course is already being given along with the course in more extended algebra in a number of our high schools.

To the ultra-conservative professor of mathematics, this interest in the calculus on the part of outsiders may seem like trespassing upon sacred ground. And, although the lover of pure mathematics is open-minded, he will ask whether a study of the calculus can be of much benefit unless the subject is approached from the scientific point of view. Therefore, it may be well to recall that our usual method of approach to the subject through limits is a relatively recent one so far as rigor is concerned. This method depends upon the thorough work on limits and infinitesimals by Carnot (1797) and the obtaining of the fractional form of the derivative by the method of limits as given by Cauchy (*c.* 1821). Yet, as we well know, wonders had been accomplished with the calculus long before this scientific method had become traditional.

It may, therefore, be of interest to consider a noteworthy attempt at classification of simple integral forms by Johann Bernoulli, the first great expositor of the calculus. Although his work did not appear in book form, his manuscripts written in 1691–1692 form the first textbook in integral calculus.[6]

[1] S. J. Willis, *A Short Course in Elementary Mathematics and Their Application to Wireless Telegraphy*, London, 1917.

[2] J. W. Mellor, *Higher Mathematics for Students of Chemistry and Physics*, New York, 1919.

[3] *Flight*, September 2, 1920 (Official organ of the royal aero group of the United Kingdom); *Aeronautics*, September 2, 1920, London; *The Aeronautic Journal*, September, 1920; *Aircraft Engineering*, January, 1920, London.

[4] J. C. Brown, *Curricula in Mathematics*, United States Bureau of Education Bulletin, No. 45, 1914.

[5] *Elective Courses in Mathematics*, The National Committee on Mathematical Requirements, March, 1921.

[6] Johann Bernoulli lived from 1667 to 1748. His *Lectiones mathamaticae de methodo integralium* was published in the third volume of *Opera Omnia*, Lausannæ & Genevæ, 1742, pp. 385–558, but the differential part of his manuscript was not published. It is stated that this was because the ground had been fully covered by the Marquis de l'Hospital (1661–1704) in his *Analyse des*

His first lecture concerns the nature and the calculation of the integral. His method is that of the inverse of differentiation. He says that it is known that $dx$ is the differential of $x$, and $xdx$ the differential of $\frac{1}{2}xx$ or $\frac{1}{2}xx \pm$ a constant, and so on. Then

$adx$ is the differential of $ax \pm$ a constant,
$axdx$ is the differential of $\frac{1}{2}axx \pm$ a constant,
$axxdx$ is the differential of $\frac{1}{3}ax^3 \pm$ a constant,
$ax^3dx$ is the differential of $\frac{1}{4}ax^4 \pm$ a constant,

and so on.[1]

From this he deduces the general formula that $ax^pdx$ is the differential of $ax^{p+1}/(p+1)$.

In his second lecture, Bernoulli considers the plane surface as divided into an infinite number of parts, each of which may be regarded as the differential of the surface. If we have the integral of this differential, that is, the sum of these parts, then will we also know the desired area. Those infinitely little parts of the plane surface can be thought of as obtained in different ways according to the most convenient subdivision of the figure. For example, plane surfaces would be directly divided, as is the custom, by means of parallel lines, as in Fig. 1,

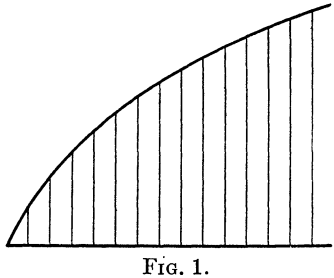


FIG. 1.

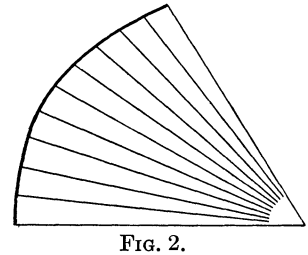


FIG. 2.

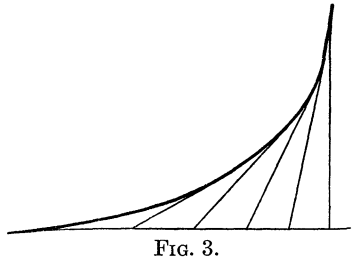


FIG. 3.

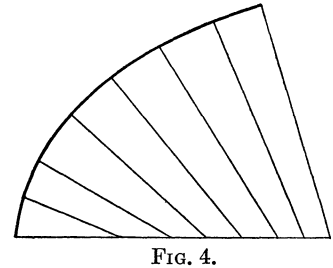


FIG. 4.

or by means of an infinite number of straight lines which are concurrent, as in Fig. 2, or by means of an infinite number of tangents, as in Fig. 3, or by means of an infinite number of normals to the curve, as in Fig. 4.

*infiniment petits*, Paris, 1696. There has been some controversy as to whether the latter book may not have really been based upon the manuscript of Bernoulli more fully than would be inferred from the general acknowledgment in the Preface. The manuscript of this part of Bernoulli's work written in Latin, has recently been found in the library of the University of Basel and is reproduced in full, with an Introduction by Paul Schaftheitlin, in *Verhandlungen der Naturforschenden Gesellschaft in Basel*, vol. 34, 1922, pp. 1–32, with four plates of figures. See also Paul Schaftheitlin, "Johann Bernoulli's Differentialrechnung," *ibid.*, vol. 32, 1921, pp. 230–235—EDITOR.

[1] *Opera Omnia*, vol. 3, pp. 387–388.

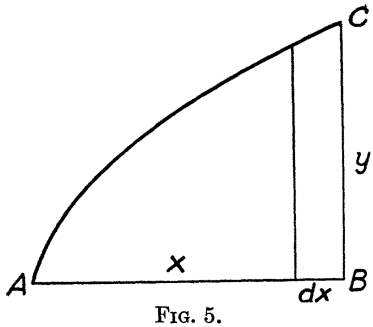


FIG. 5.

If the divisions of the surface are parallel, and if $x$ is the abscissa and $y$ is the ordinate, the differential of the surface will be $ydx$; namely, the rectangle formed by the ordinate and the differential of the abscissa. If $AC$ is the given curve, then $y$ will have a given ratio to $x$, so that it is expressed in $x$ alone. Suppose that $AC$ is a parabola, then $ax = y^2$ or $y = \sqrt{(ax)}$. The integral of this, which is $\frac{2}{3}x\sqrt{(ax)}$ or $\frac{2}{3}xy$, is the area desired.[1] See Fig. 5.

This method of Bernoulli's for finding the area is simple, and is satisfactory for elementary work, although the idea of limits involved is taken care of largely by intuition.

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## AMONG MY AUTOGRAPHS.

By DAVID EUGENE SMITH, Columbia University.

### 29. LEGENDRE AND CAUCHY SPONSOR ABEL.

In his work on Abel,[2] M. Ch. Lucas de Pesloüan gives, in a supplementary note (p. 144), the beginning of a memoir which, as he says, "probablement fut lu par Fourier en séance publique, le 30 Octobre 1826, à l'Institut, et que l'on ne comprit pas," stating in the text (p. 55) that it is said to be a development of the addition theorem.

In volume 2 of Abel's *Oeuvres Complètes* (1839) the editor, B. Holmboe, has the statement that, much as he had wished to publish in his edition the memoir presented by Abel about the close of the year 1826, "tous les efforts pour obtenir une copie de ce mémoire ont eté infructueux jusqu'à présent."

In the *Mémoires présentés par divers savants à l'Académie Royale des Sciences de l'Institut de France,* published at Paris two years later (1841),[3] however, is the very memoir referred to, with the title: "Mémoire sur une propriété générale d'une classe très étendue de Fonctions Transcendantes, par M. N. H. Abel, Norvégien, présenté à l'Académie le 30 Octobre, 1826." On the last page (264) there is a note by Libri, the historian of mathematics, as follows: "L'Académie m'ayant fait l'honneur de me charger de surveiller l'impression de ce Mémoire, je me suis appliqué à corriger, autant que possible, les fautes d'impression," with apologies for not being able to do as well as could have been done had he been able to see the original manuscript. Indeed, when the second edition of Abel's work was published at Christiania (1881), under the editorship of Sylow and Lie, there was inserted a note on this memoir (volume 2, p. 294) to the effect that Lie

[1] *Ibid.*, pp. 394–395.
[2] N. H. Abel, *Sa vie et son oeuvre*, Paris, 1906.
[3] Vol. 7, pp. 176–264.